

## Physics 1C Discussion (Week 6)

### Problem 1

**32.40 •• CP** A circular wire loop has a radius of 7.50 cm. A sinusoidal electromagnetic plane wave traveling in air passes through the loop, with the direction of the magnetic field of the wave perpendicular to the plane of the loop. The intensity of the wave at the location of the loop is  $0.0275 \text{ W/m}^2$ , and the wavelength of the wave is 6.90 m. What is the maximum emf induced in the loop?

We start by following the hints and finding an expression for  $B_{\max}$ :

$$I = \frac{E_{\max}^2}{2\mu_0 c}. \quad (1)$$

There are multiple correct ways to write this formula. To convert  $E_{\max}$  to  $B_{\max}$ , we write

$$E_{\max} = cB_{\max}, \quad (2)$$

and therefore,

$$I = \frac{c}{2\mu_0} B_{\max}^2 \implies B_{\max} = \sqrt{\frac{2\mu_0 I}{c}} \approx 1.52 \times 10^{-8} \text{ T} \quad (3)$$

Given this, because the electromagnetic wave varies sinusoidally, we can write:

$$|\vec{B}(t)| = B_{\max} \cos(\omega t) = \sqrt{\frac{2\mu_0 I}{c}} \cos(\omega t) \quad (4)$$

To put this in terms of known quantities,  $\omega$  is the angular frequency, so,

$$\omega = 2\pi\nu = 2\pi \frac{c}{\lambda} \approx 2.73 \times 10^8 \text{ rads/s}. \quad (5)$$

Plugging this back in,

$$|\vec{B}(t)| = \sqrt{\frac{2\mu_0 I}{c}} \cos\left(2\pi \frac{c}{\lambda} t\right). \quad (6)$$

Finally, to calculate the maximum emf, we can say that  $B(t)$  is roughly constant through the loop, such that:

$$\begin{aligned} \mathcal{E}_{\max} &= -\left. \frac{d\Phi_B(t)}{dt} \right|_{\max} \\ &= \left[ -\frac{d}{dt} \int \vec{B}(t) \cdot d\vec{A} \right]_{\max} \\ &= \left[ -\frac{dB}{dt} \int dA \right]_{\max} \\ &= \left[ \sqrt{\frac{2\mu_0 I}{c}} 2\pi \frac{c}{\lambda} \sin\left(2\pi \frac{c}{\lambda} t\right) \cdot \pi r^2 \right]_{\max} \end{aligned} \quad (7)$$

The maximum of sin is 1, and so,

$$\mathcal{E}_{\max} = \sqrt{\frac{2\mu_0 I}{c}} 2\pi \frac{c}{\lambda} \cdot \pi r^2 = \sqrt{8c\mu_0 I} \frac{\pi^2 r^2}{\lambda} \approx 7.32 \times 10^{-2} \text{ V} \quad (8)$$